LAU
الجامعة اللبنانية الأميركية
Lebanese American University

LAU
School of Engineering
Department of Electrical and Computer Engineering

ELE 305: Introduction to Electrical Engineering

Exam 1 – Fall 2016

Duration: **1 hour 30 minutes**
Date: 8/10/2016
Start Time: 2:00 pm

Dr. Harag Margossian

Name: ______________________________ **ID#:** ____________________

INSTRUCTIONS:

- Answer each of the following questions in the space provided.
- You can use both sides of the sheets for answers.
- Solutions written outside this booklet will not be graded.
- This is a closed-book exam
- Programmable calculators and smart devices are not allowed.
- The number of points for each question is specified next to it.
- The total number of points is 100.

1	2	3	4	5	Total
/10	/15	/20	/25	/30	/100

Question 1 (10 points)

The current entering an element is shown in Figure 1. Find the charge that enters the element in the time interval 0<t<30s

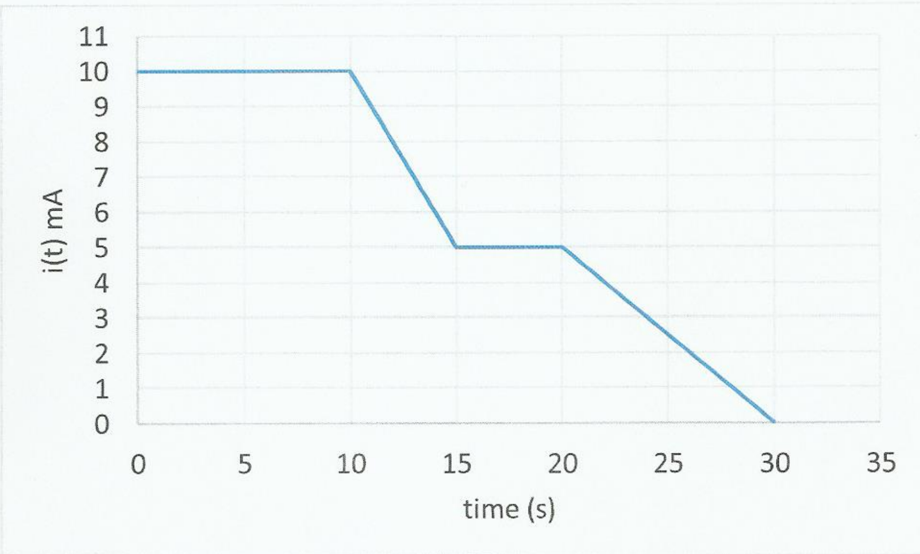


Figure 1

According to the figure: $i(t)\ (mA) = \begin{cases} 10 & 0 \le t \le 10 \\ -t+20 & 10 \le t \le 15 \\ 5 & 15 \le t \le 20 \\ -\frac{t}{2}+15 & 20 \le t \le 30 \end{cases}$

$i(t) = \frac{dq(t)}{dt} \Rightarrow q(t)\ (mC) = \int_{t_0}^{t} i(t)\,dt + q(t_0)$

• $0 \le t \le 10$: $q(t) = \int_0^t i(t)\,dt + q(0) = \int_0^t 10\,dt + 0 = 10t$

• $10 \le t \le 15$: $q(t) = \int_{10}^t i(t)\,dt + q(10) = \int_{10}^t (-t+20)\,dt + q(10) = \left[-\frac{t^2}{2} + 20t\right]_{10}^t + q(10)$

$\boxed{q(10) = 10 \times 10 = 100\ mC}$

$\Rightarrow q(t) = -\frac{t^2}{2} + 20t + 50 - 200 + 100 = -\frac{t^2}{2} + 20t - 50$

• $15 \le t \le 20$: $q(t) = \int_{15}^t i(t)\,dt + q(15) = [5t]_{15}^t + q(15) = 5t - 75 + \frac{225}{2} = 5t + \frac{75}{2}$

$\boxed{q(15) = \frac{-(15)^2}{2} + 20 \times 15 - 50 =} -\frac{225}{2} + 300 - 50 = \frac{225}{2}$

• $20 \le t \le 30$: $q(t) = \int_{20}^t i(t)\,dt + q(20) = \int_{20}^t \left(-\frac{t}{2} + 15\right) + q(20) = \left(-\frac{t^2}{4} + 15t\right)_{20}^t + q(20)$

$= \text{— — — —}$

$q(20) = 5 \times 20 + \frac{75}{2} = \frac{275}{2}$

Question 2 (15 points)

Find R_{AB} in the network in Figure 2.

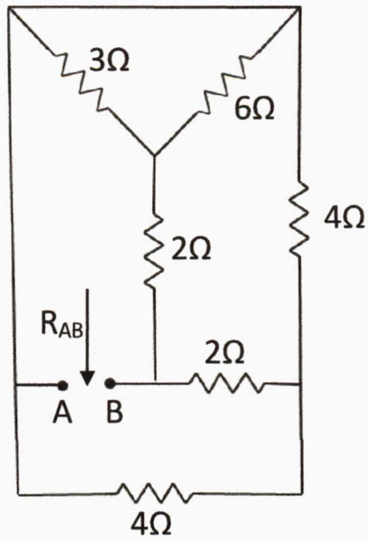
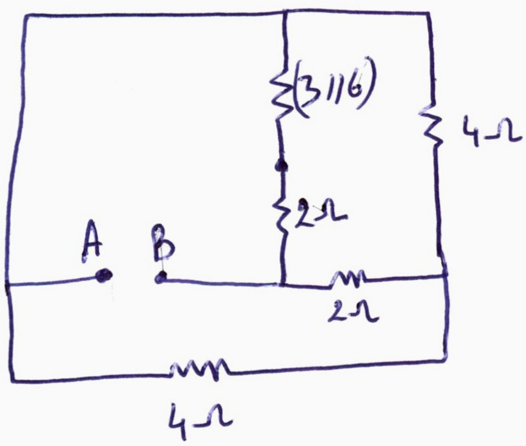
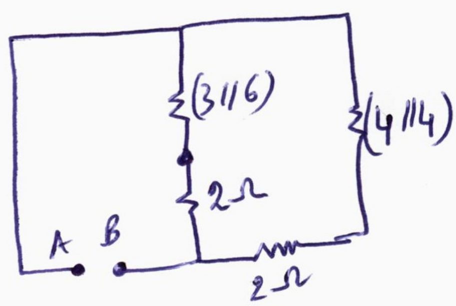


Figure 2

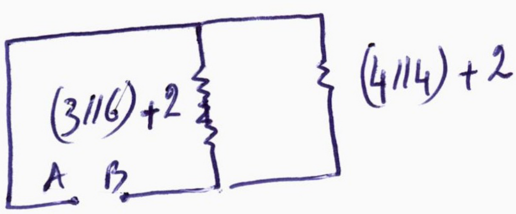


The two (4Ω) resistances are in //

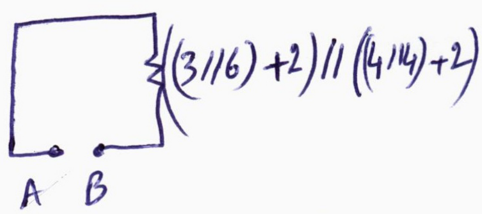
⟺



⟺



⟺



$R_{AB} = (2+2) \,//\, (2+2)$

$= 4 \,//\, 4 = 2\,\Omega$

Question 3 (20 points)

Use superposition to find V_o in the network in Figure 3.

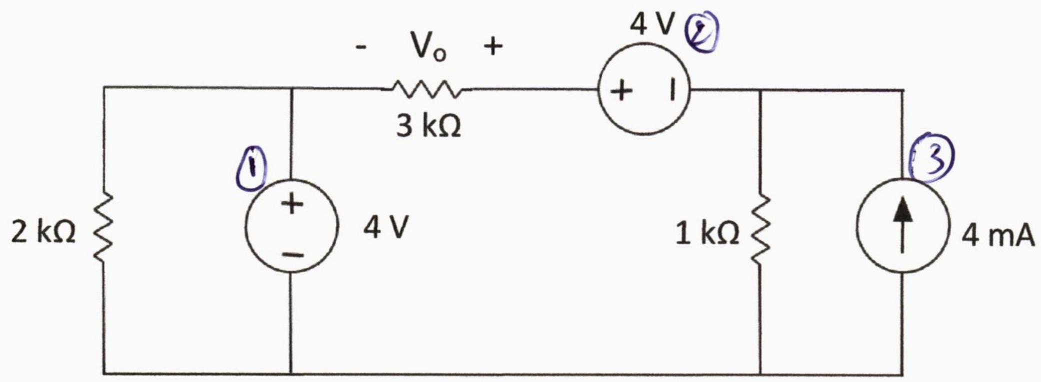
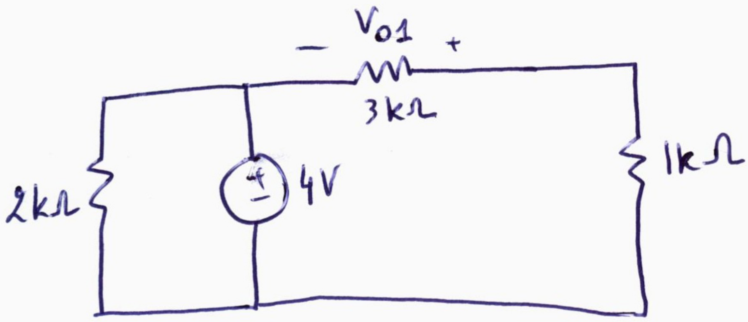


Figure 3

phase I

1 ON , 2 OFF, 3 OFF



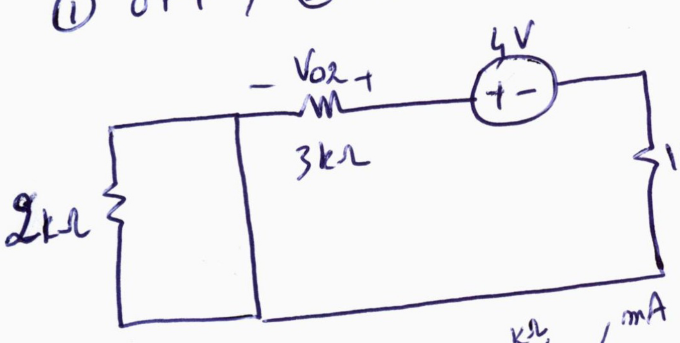
Voltage divider ⟹

$V_{o1} = -4 \times \dfrac{3}{3+1}$ — the ratio in here is $\dfrac{k\Omega}{k\Omega}$, so No problem

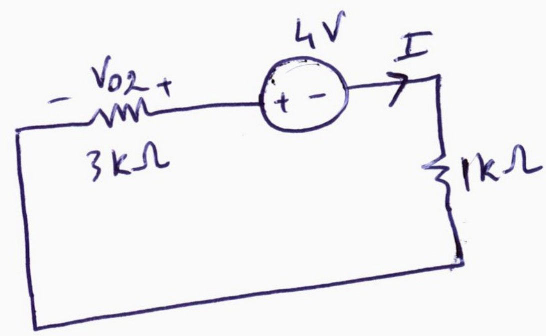
$= -4 \times \dfrac{3}{4} = -3\,V$

phase II

1 OFF , 2 ON , 3 OFF



⟺



KVL ⟹ $\overset{k\Omega}{1} \times \overset{mA}{I} - V_{o2} + 4 = 0$

But $V_{o2} = -3 \times I$

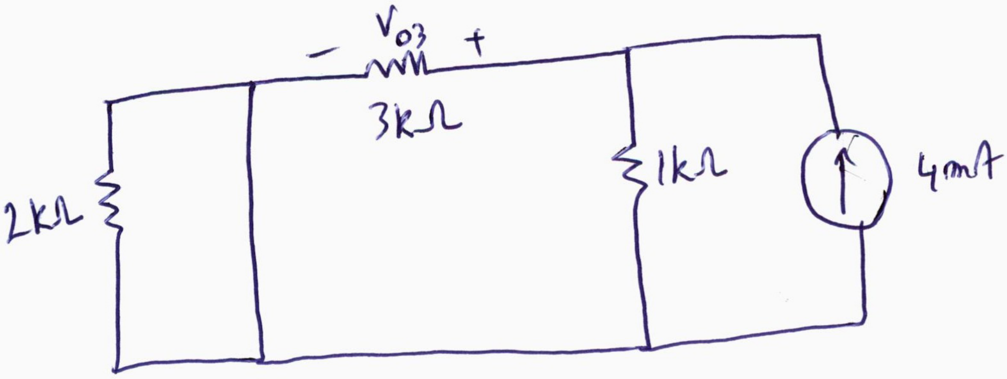
$\Rightarrow I + 3I + 4 = 0$

$\Rightarrow 4I = -4 \Rightarrow I = -1\,mA$

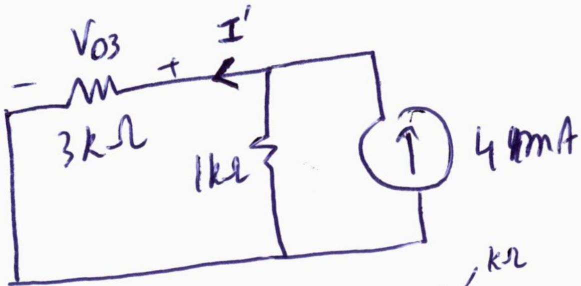
Then $V_{o2} = -3 \times I = -\overset{k\Omega}{3} \times \overset{mA}{(-1)} = 3\,V$

phase III

① OFF , ② OFF, ③ ON



$\Longleftrightarrow$



Current divider: $I' = \underset{mA}{4} \times \frac{1^{k\Omega}}{3+1_{k\Omega}} = 1\ mA$

$V_{03} = +\ \underset{k\Omega}{3} \times I' = 3 \times \underset{mA}{1} = 3V$

Finally, $V_0 = V_{01} + V_{02} + V_{03} = -3 + 3 + 3 = 3V$

Question 4 (25 points)

Use source transformation to find I_o in the network in Figure 4.

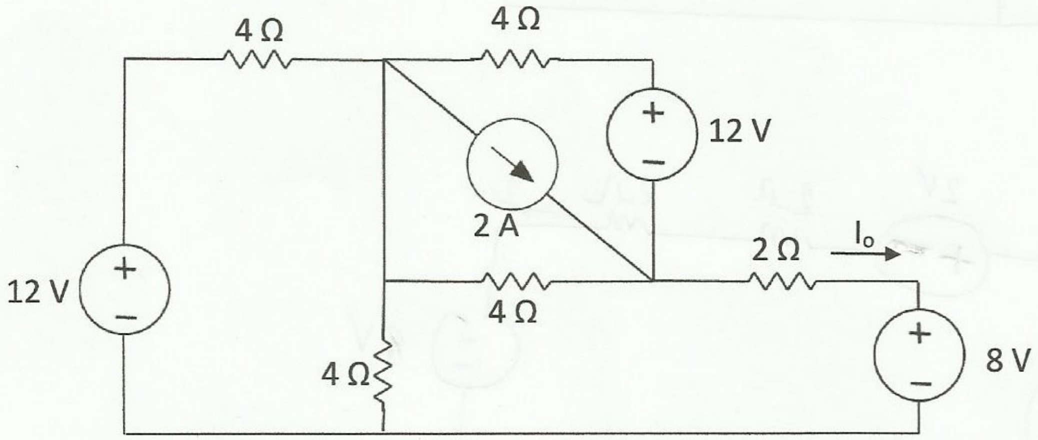
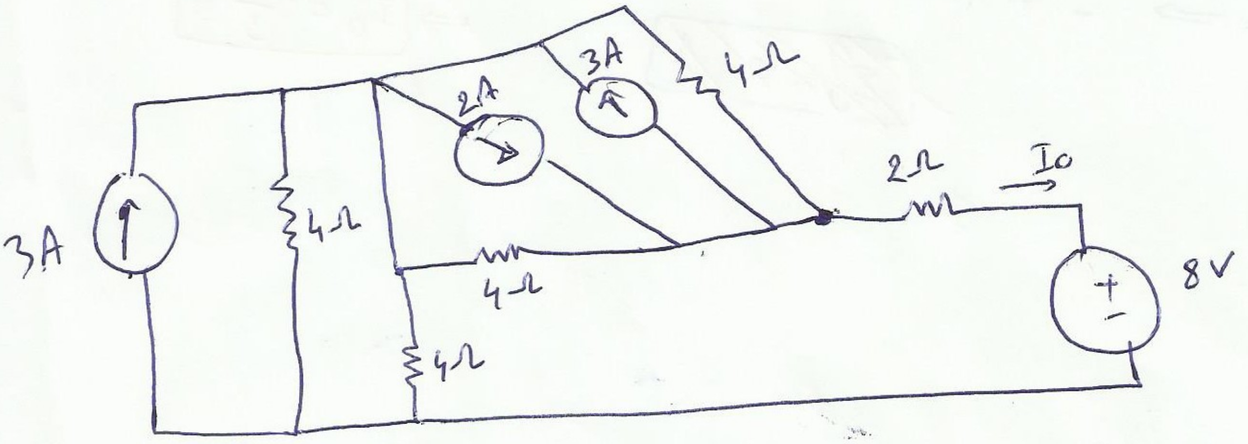
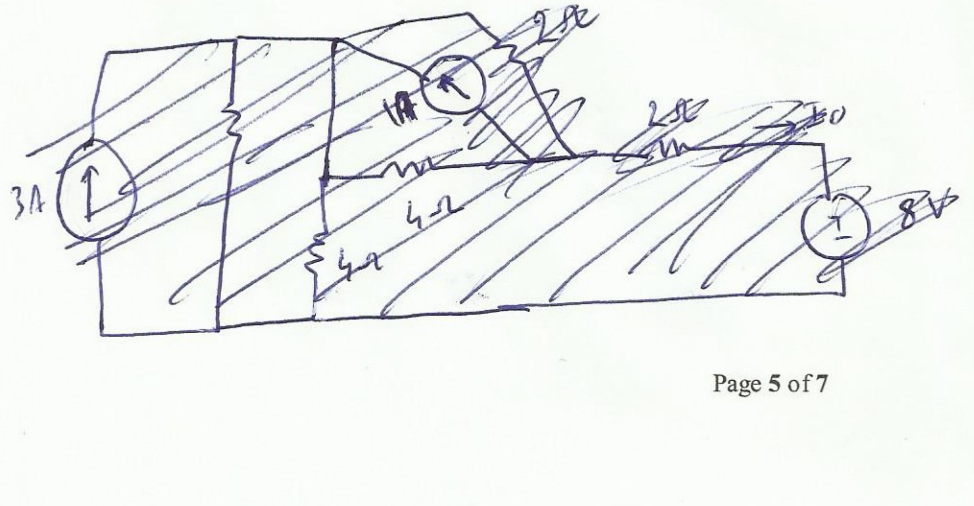
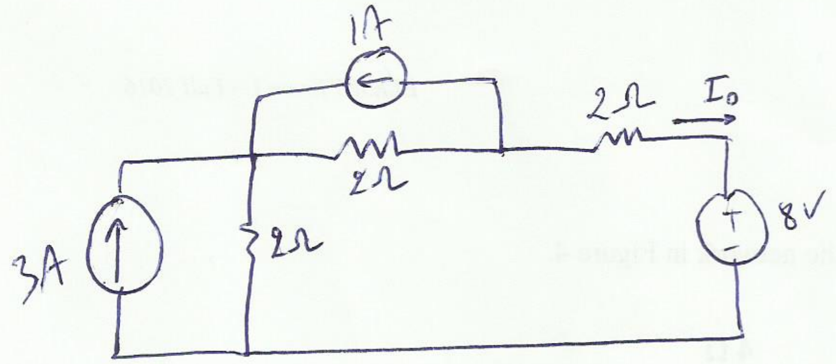
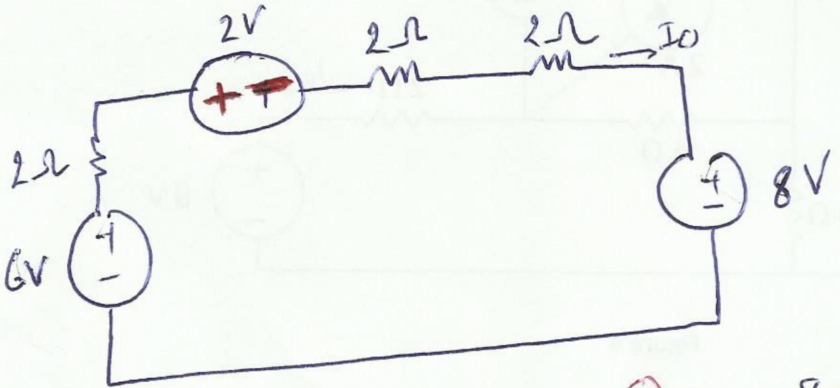


Figure 4









KVL $\Longrightarrow$ $-6 + 6I_0 + 2 + 8 = 0 \Longrightarrow 6I_0 = -4$

$\cancel{\boxed{I_0 = 0\ A}}$ $\Longrightarrow \boxed{I_0 = -\frac{2}{3}\ A}$

Question 5 (30 points)

Consider the network in Figure 5.

a. Find the thevenin equivalent of Circuit A as seen by the rest of the network.
b. Calculate the maximum power that can be transferred to R_L and the value of R_L for which this happens.

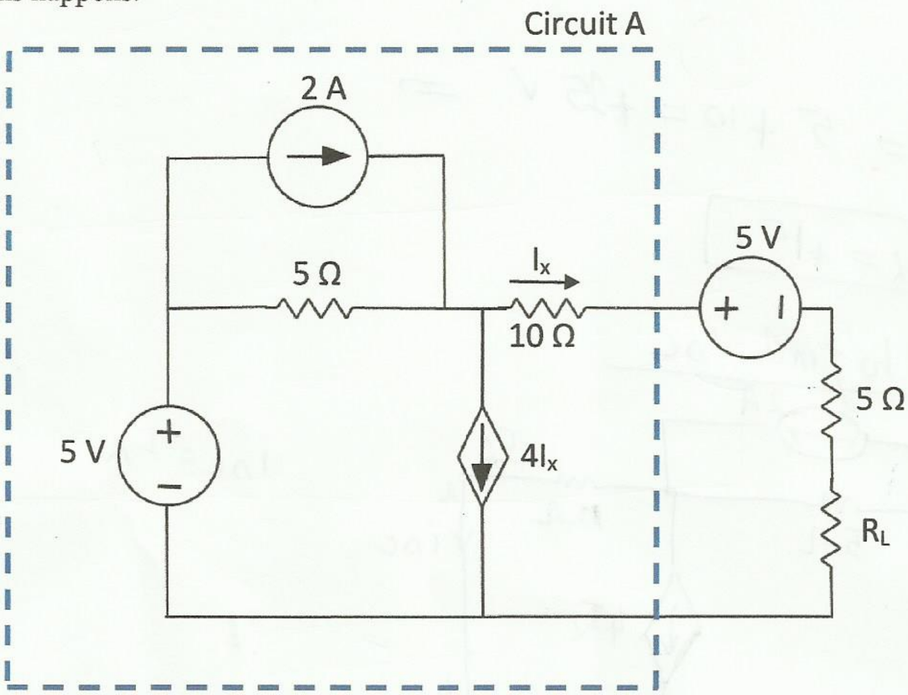
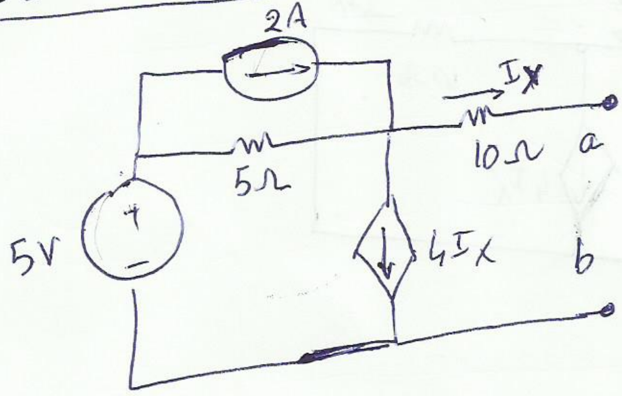


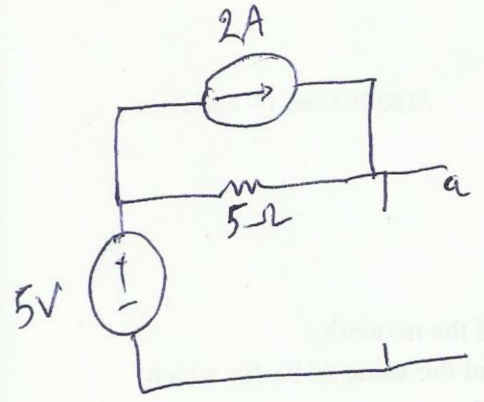
Figure 5

We cut circuit A: (we remove the remaining part from the circuit)

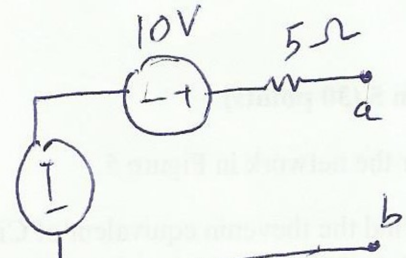


$V_{th} = V_{ab} = V_{oc}$

since open circuit $\Rightarrow I_x = 0\ A \Rightarrow 4I_x = 0\ A \Rightarrow$ we can open the branch of the $(4I_x)$ dependent current source and remove the 10Ω resistance



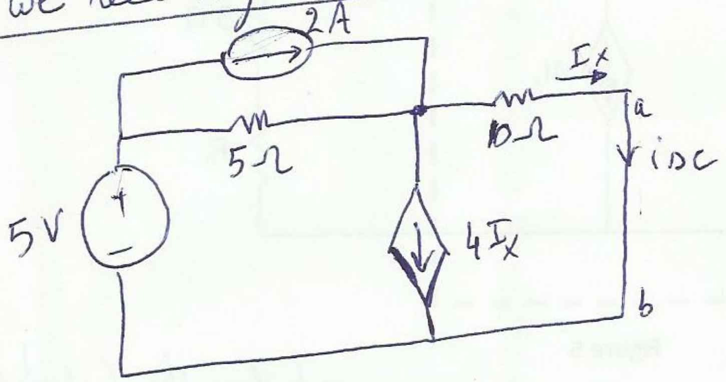
Source $\Longleftrightarrow$ transformation



(KVL) $V_{ab} = 5 + 10 = +15\text{ V} \Rightarrow$

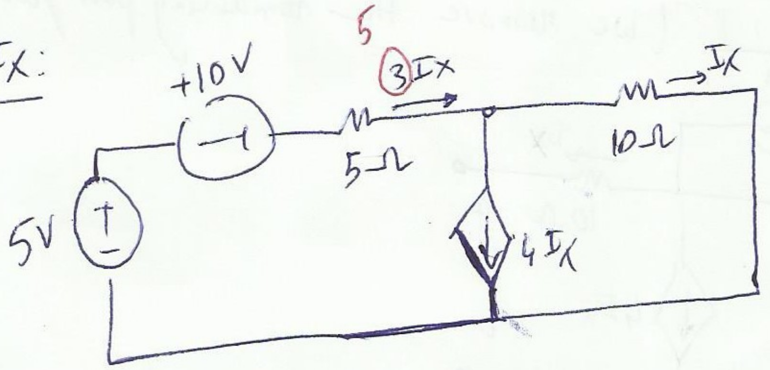
$\boxed{V_{th} = +15\text{ V}}$

Now, we need to find i_{sc}:



$i_{sc} = I_x$

Let us find I_x:



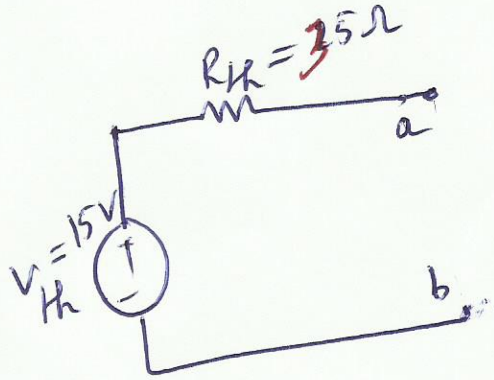
Apply KVL to the outer loop:

$-5 - 10 + 5 \times 5 I_x + 10 I_x = 0 \Rightarrow$

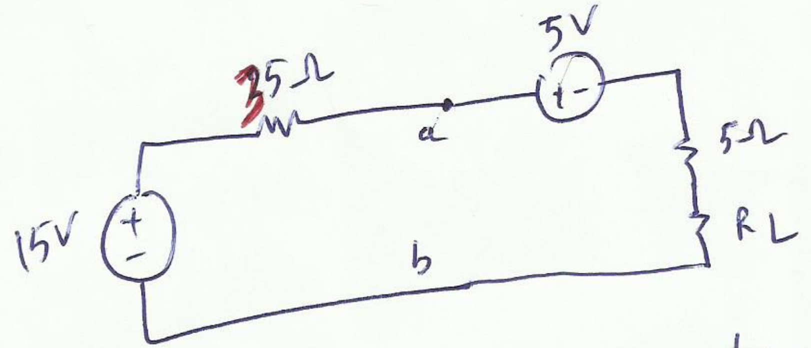
$35 I_x = 15 \Rightarrow I_x = \frac{15}{35} = \frac{3}{7}\text{ A} \Rightarrow i_{sc} = \frac{3}{7}\text{ A}$

Then $R_{Th} = \frac{V_{th}}{i_{sc}} = \frac{15\text{V}}{3/7} = 35\,\Omega$

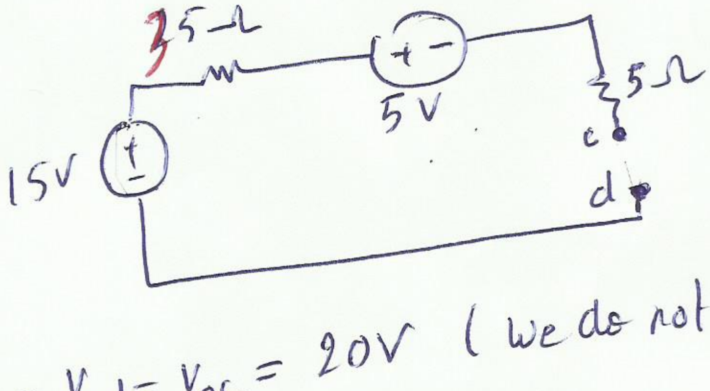
The Thevenin Equivalent of the circuit (A) as seen by the remaining elements is:



b. the equivalent circuit looks like



In order to ensure a max power transfer to R_L, we need to find the R_{th} of the thevenin equivalent circuit as seen by R_L:



→ $V'_{th} = V_{cd} = V_{oc} = 20V$ (we do not need it)

→ to find R_{th}, zero the voltage sources and find $R_{eq} = R_{cd}$.

we obtain $R_{cd} = \cancel{30}\,40\,\Omega$. Then $\boxed{R_L = 40\,\Omega}$

$= R_{eq} = R'_{th}$

For a max power transfer to R_L